1) Use the definition to obtain \( \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h+y} - \sqrt{x+y}}{h} \). Next multiply the numerator and the denominator by the conjugate and the expression simplifies as

\[
\lim_{h \to 0} \frac{1}{\sqrt{x+h+y} + \sqrt{x+y}} = \frac{1}{2\sqrt{x+y}}
\]

2) a) First find the directional vector PQ by using the formula terminal – initial = \((-1, 3, 2)\). Then divide by its magnitude to make a unit vector: \( u = \left(-\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right) \) Then compute the gradient of \( f \), evaluate it at \( P \): we have \( \nabla f = \left\langle 8x, -2y, \frac{16}{1+z^2} \right\rangle \) \( (4, -2, 1) \) \( = \left\langle 32, 4, 8 \right\rangle \) Your answer can now be obtained by \( \nabla f \cdot u = -\frac{4}{\sqrt{14}} \)

b) Increases most rapidly in the direction \( \frac{\nabla f}{|\nabla f|} = \frac{1}{\sqrt{32^2 + 4^2 + 8^2}} \cdot \left\langle 32, 4, 8 \right\rangle \) (dividing by its magnitude is optional: you may leave your answer as \( \nabla f = \left\langle 32, 4, 8 \right\rangle \) ), decreases most rapidly in the direction \( \frac{\nabla f}{|\nabla f|} = \frac{1}{\sqrt{32^2 + 4^2 + 8^2}} \cdot \left\langle -32, 4, 8 \right\rangle \), or \( -\nabla f = \left\langle -32, 4, 8 \right\rangle \) the magnitude of the change is \( |\nabla f| = \sqrt{32^2 + 4^2 + 8^2} \)

3) A) \( k = 0 \): cone: \( x^2 + y^2 = z^2 \) No, a 4 dim'l graph cannot be sketched

b) The domain of \( f(x, y) = \sqrt{4 - x^2 - 4y^2} \to 4 - x^2 - 4y^2 \geq 0 \to x^2 + 4y^2 \leq 4 \) This is an ellipse with x-intercepts 2 and -2, y-intercepts 1 and -1.

4) 8)a) Use the squeeze theorem: \( 0 \leq \frac{x^4}{x^2 + y^2} \leq \frac{x^4}{x^2} = x^2 \) Since \( x^2 \) approaches 0, the limit is 0  b) The limit does not exist: The path through the positive x-axis produces 2, whereas the path through the positive y-axis produces -1/2
5) $V = \pi r^2 h \rightarrow dV = 2\pi rh \, dr + \pi^2 \, dh = 2\pi(3)(8)(0.05) + 2\pi(9)(0.05)$

6) \[ \frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial r} = \frac{\partial z}{\partial x} (2r) + \frac{\partial z}{\partial y} (-1) \]

\[ \frac{\partial^2 z}{\partial r \partial s} = \frac{\partial}{\partial r} \left( \frac{\partial z}{\partial r} \right) = \frac{\partial}{\partial r} \left( \frac{\partial z}{\partial x} (2r) + \frac{\partial z}{\partial y} (-1) \right) = \frac{\partial}{\partial r} \left( \frac{\partial z}{\partial x} (2s) - \frac{\partial z}{\partial r} \frac{\partial z}{\partial y} \right) = \]

\[ = 2s \left[ \frac{\partial^2 z}{\partial x^2} \frac{\partial z}{\partial r} + \frac{\partial^2 z}{\partial x \partial y} \frac{\partial z}{\partial y} \right] - \left[ \frac{\partial^2 z}{\partial x \partial y} \frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} \frac{\partial z}{\partial y} \right] =
\]

\[ = 4sr \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} (2s - 2r) - \frac{\partial^2 z}{\partial y^2} (1) \]

7) \[ \text{Use implicit differentiation: treat } x \text{ as a constant, multiply by } \frac{\partial z}{\partial y} \text{ after differentiating with respect to } z: \]

\[ x^2 y + z \tan^{-1} \frac{y}{x} + zy^2 = 1 \rightarrow x^2 + \frac{\partial z}{\partial y} \tan^{-1} \frac{y}{x} \left(1 + \left(\frac{y}{x}\right)^2 \right) x + \frac{\partial z}{\partial y} y^2 + 2zy = 0 \]

\[ \rightarrow \frac{\partial z}{\partial y} = \frac{-x^2 - z \left(1 + \left(\frac{y}{x}\right)^2 \right) x}{\tan^{-1} \frac{y}{x} + y^2} \]

8) \[ \text{A) Ln function is defined when the inside is positive: b) } \ln 3 = \ln(x - y) \rightarrow 3 = x - y \]

9) \[ \text{A)} \lim_{h \to 0} \frac{f(1, 2 + h) - f(1,2)}{h} = \lim_{h \to 0} \frac{(3)(1)(2 + h) - 3(1)(2)}{h} = 3 \text{ b)} \]

\[ \lim_{s \to 0} \frac{f(x + su_1, y + su_2) - f(x, y)}{s} = \lim_{s \to 0} \frac{f(1 + (-0.6)s, 2 + (0.8)s) - f(1, 2)}{s} = \lim_{s \to 0} \frac{3(1 - 0.6s)(2 + 0.8s) + (1 - 0.6s) - 3(1)(2) - 1}{s} = \lim_{s \to 0} \frac{6 + 2.4s - 3.6s - 1.44s^2 + 1 - 0.6s - 6 - 1}{s} = -1.8 \]

pg. 2
10) First make a tree diagram

\[ \frac{\partial I}{\partial t} = \frac{\partial I}{\partial V} \frac{dV}{dt} + \frac{\partial I}{\partial R} \frac{dR}{dt} \]

Since \( I = \frac{V}{R}, \quad \frac{\partial I}{\partial V} = \frac{1}{R}, \quad \frac{\partial I}{\partial R} = -\frac{V}{R^2} \). Since \( V=80 \) and \( R =40 \),

\[ \frac{\partial I}{\partial V} = \frac{1}{40}, \quad \frac{\partial I}{\partial R} = -\frac{80}{40^2} = -\frac{1}{20}. \]

It is given in the problem that \( \frac{dV}{dt} = 5, \quad \frac{dR}{dt} = -2 \). Therefore,

\[ \frac{\partial I}{\partial t} = \frac{\partial I}{\partial V} \frac{dV}{dt} + \frac{\partial I}{\partial R} \frac{dR}{dt} = \frac{1}{40} (5) + \frac{1}{20} (-2) = \frac{1}{40} \text{ amp/min.} \]

11) \( f \) decreases most rapidly in the direction

\[ -\frac{\nabla f}{|\nabla f|}\bigg|_{(1,0)} = \frac{\langle ye^{xy}, xe^{xy} + 1 \rangle}{|\nabla f|}\bigg|_{(1,0)} = -\left(\frac{1}{\sqrt{5}} <0,2> \right) = <0 - \frac{2}{\sqrt{5}}>, \]

the magnitude of the change is \( |\nabla f| = \frac{2}{\sqrt{5}} \)

12) \( \frac{\partial w}{\partial t} = \frac{\partial w}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial w}{\partial v} \frac{\partial v}{\partial t} \)

13) A) It is a “prism” in the first octant: bounded on the top by \( z = 2x \), the side by \( x + y = 6 \), a vertical Plane

A) \( \iiint_{0}^{6} 2x \, dydz \, dx \) B) the projection onto the yz-plane can be found by eliminating \( x \) from \( z = 2x \)

and \( x + y = 6 \):

\[ \iiint_{0}^{6} 2 \, dydz \, dx \]

\[ \iiint_{0}^{6} x \, dydz \, dx \]
14) the domain is all real ordered pairs ($\mathbb{R}^2$) a) not bounded b) no boundary points c) open and closed, (clopen) d) sketch the level curve $f(x, y) = 2 \rightarrow e^{x^2+y^2} = 2 \rightarrow x^2 + y^2 = \ln 2$. This is a circle of radius $\ln 2$.

15) $\lim_{s \to 0} \frac{f(x + su_1, y + su_2) - f(x, y)}{s} = \lim_{s \to 0} \frac{f(2 + (0.6)s, 1 + (0.8)s) - f(2, 1)}{s}$

$= \lim_{s \to 0} \frac{(2 + 0.6s)^2 + 4(2 + 0.6s)(1 + 0.8s) - 2^2 - 4(2)(1)}{s}$

$= \lim_{s \to 0} \frac{4 + 2.4s + 0.36s^2 + 8 + 8.8s + 0.96s^2 - 4 - 8}{s} = 11.2$

16) a) $\nabla f|_{(1,3)} = \langle 2x + 4, y, 4x > \mid_{(1,3)} = \langle 14, 4 >$ Increase most rapidly in the direction $\frac{\nabla f}{|\nabla f|} = \frac{1}{\sqrt{14^2 + 4^2}} < 14, 4 >$, decreases most rapidly in the direction $-$ $\frac{\nabla f}{|\nabla f|} = -(\frac{1}{\sqrt{14^2 + 4^2}} < 14, 4 >)$, (dividing by its magnitude is optional: you may leave your answer as $\nabla f = < 14, 4 >$) the magnitude of the change is $|\nabla f| = \sqrt{14^2 + 4^2}$

b) The directions of zero change are perpendicular to the gradient: for a 2 dim'l vector, switch x and y, multiply one of the components by -1. Zero change: $\frac{< 4, -14 >}{\sqrt{212}} = \frac{-4, 14}{\sqrt{212}}$

17) a) $\lim_{(x,y) \to 0} \frac{x^2}{x^2 + y^2}$ does not exist: First take $x = y$, next take $x = 0$. These two paths produce different limits.

b) $\lim_{(x,y) \to 0} \frac{x^4}{x^2 + y^2}$ exist, since by using the squeeze theorem, $0 \leq \frac{x^4}{x^2 + y^2} \leq \frac{x^4}{x^2} = x^2$. Since the right side goes to 0, $\lim_{(x,y) \to 0} \frac{x^4}{x^2 + y^2} = 0$

c) $\lim_{(x,y) \to 0} 2y^8 + 2x^4$ does not exist: First try the path $y = x^2$ which produces the limit of $\frac{3}{2}$. Next try the path through the positive x axis $y = 0$ which produces 0.

18) a) $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial w}{\partial v} \frac{\partial v}{\partial x} = \frac{\partial w}{\partial u} (3y) + \frac{\partial w}{\partial v} (1)$,

$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial x} \right) = \frac{\partial}{\partial x} \left[ 3y + \frac{\partial w}{\partial v} \right] = 3y \left[ \frac{\partial^2 w}{\partial u^2} \frac{\partial u}{\partial x} + \frac{\partial^2 w}{\partial u v} \frac{\partial v}{\partial x} \right] + \left[ \frac{\partial w}{\partial u} \frac{\partial u}{\partial x} \right] + \left[ \frac{\partial w}{\partial v} \frac{\partial v}{\partial x} \right]$

$= 3y \left[ \frac{\partial^2 w}{\partial u^2} (3y) + \frac{\partial^2 w}{\partial u v} (1) \right] + \left[ \frac{\partial^2 w}{\partial u v} (3y) + \frac{\partial^2 w}{\partial v^2} (1) \right]$

$= 9y^2 \frac{\partial^2 w}{\partial u^2} + 6y \frac{\partial^2 w}{\partial u v} + \frac{\partial^2 w}{\partial v^2}$

$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} + \frac{\partial f}{\partial w} + \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} + \frac{\partial f}{\partial w} + \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} + \frac{\partial f}{\partial w}$

b) $\frac{\partial f}{\partial u} (1) + \frac{\partial f}{\partial v} (-1) + \frac{\partial f}{\partial w} (-1) + \frac{\partial f}{\partial u} (1) + \frac{\partial f}{\partial v} (1) + \frac{\partial f}{\partial w} (1) = 0$
19) To compute \( \frac{\partial^2 w}{\partial \theta \partial s} = \frac{\partial}{\partial \theta} \left( \frac{\partial w}{\partial s} \right) \), first compute \( \frac{\partial w}{\partial s} \) using the tree diagram.

\[
\frac{\partial w}{\partial x} (r^2) + \frac{\partial w}{\partial y} (5) \quad \text{Now compute} \quad \frac{\partial^2 w}{\partial \theta \partial s} = \frac{\partial}{\partial \theta} \left( \frac{\partial w}{\partial s} \right) = \frac{\partial}{\partial \theta} \left( \frac{\partial w}{\partial x} r^2 + \frac{\partial w}{\partial y} 5 \right)
\]

\[
= \frac{\partial^2 w}{\partial x^2} (2rs) + \frac{\partial^2 w}{\partial x \partial y} (4)(r^2) + \frac{\partial w}{\partial x} (2r) + 5(\frac{\partial^2 w}{\partial x \partial y} (2rs) + \frac{\partial^2 w}{\partial y^2} (4))
\]

\[
= \frac{\partial^2 w}{\partial x^2} (2r^3s) + \frac{\partial^2 w}{\partial x \partial y} (4r^2 + 10rs) + \frac{\partial^2 w}{\partial y^2} (20) + \frac{\partial w}{\partial x} (2r)
\]

20) Show the limit You should sketch the region: i

A) \( \int_{0}^{4} \int_{0}^{4-z} \int_{0}^{1} dz dy dx \) B) \( \int_{0}^{4} \int_{0}^{4-z} \int_{0}^{1} dx dy dz \)

21a) When \( x = y = z = 0 \), the equation becomes \( 0 = x^2 + y^2 - z^2 \rightarrow z^2 = x^2 + y^2 \). This is a cone. b) The domain (2 dimensional) is \( x^2 - y > 0 \rightarrow y < x^2 \) It is not bounded. It is open.

22) a) \( f(x, y) = \tan^{-1} \frac{y}{x} \rightarrow \nabla f = \left\langle -\frac{y}{x^2}, \frac{1}{x^2} \right\rangle \mid_{(x, y)} = \left\langle -\frac{2}{5}, -\frac{1}{5} \right\rangle \) is the direction of most rapid increase. B) The directions of zero change are perpendicular to the gradient. To make vectors that are perpendicular to \( \nabla f \), switch the two components, multiply one of the components by negative:

\[
<\frac{1}{5}, -\frac{2}{5}>, <\frac{1}{5}, -\frac{2}{5}>
\]

23) a) Domain is \( \mathbb{R}^2\setminus\{(0,0)\} \)
24) To compute $\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial t} \right)$, first compute $\frac{\partial u}{\partial t}$ using the tree diagram.

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial t}$$

$$= \frac{\partial u}{\partial x} (5) + \frac{\partial u}{\partial y} (2s)$$

Now compute $\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial t} \right)$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \frac{\partial x}{\partial t} + \frac{\partial^2 u}{\partial x \partial y} \frac{\partial x}{\partial t} \frac{\partial y}{\partial t} + 2s \left( \frac{\partial^2 u}{\partial x \partial y} \frac{\partial x}{\partial t} + \frac{\partial^2 u}{\partial y^2} \frac{\partial y}{\partial t} \right)$$

$$= 5 \left( \frac{\partial^2 u}{\partial x^2} (5) + \frac{\partial^2 u}{\partial x \partial y} (2s) \right) + 2s \left( \frac{\partial^2 u}{\partial x \partial y} (5) + \frac{\partial^2 u}{\partial y^2} (2s) \right)$$

$$= 25 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} (20s) + \frac{\partial^2 u}{\partial y^2} (4s^2)$$

25) To find the domain, recall that the domain of $\ln x$ is $x > 0$. $f(x, y) = y \ln x^2$, the argument (inside expression) is never negative, but it is 0 if $x = 0$. Thus the domain is $\{(x, y) : x \neq 0\}$.

26) First set up the volume function. Then differentiate it with respect to $t$ using the chain rule.

$$V = abc \rightarrow \frac{dV}{dt} = \frac{\partial V}{\partial a} \frac{da}{dt} + \frac{\partial V}{\partial b} \frac{db}{dt} + \frac{\partial V}{\partial c} \frac{dc}{dt} = bc(2) + (ac)(2) + ab(2)$$

Now substitute $a = 2$, $b=3$, $c=4$.

$$\frac{dV}{dt} = (3)(4)(2) + (2)(4)(2) + (2)(3)(2) = 52 \text{ m/sec}$$

27) a) Use the squeeze theorem. You must first take the absolute value of the function since the function values may become negative:

$$0 \leq \left| \frac{2x^2 y^3}{x^4 + y^2} \right| \leq \left| \frac{2x^2 y^3}{y^2} \right| \leq |2x^2 y| \quad \text{and} \quad |2x^2 y| \rightarrow 0 \text{ as } (x, y) \rightarrow (0,0).$$

Thus $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2 y^3}{x^4 + y^2} = 0$ by the squeeze theorem.
b) \[ \lim_{(x,y)\to(2,1)} \frac{x^2 - y^2 - 3}{x - 2} \text{ does not exist: first let } x=2y. \text{ Then } \lim_{(x,y)\to(2,1)} \frac{x^2 - y^2 - 3}{x - 2} = \]

\[ \lim_{(x,y)\to(2,1)} \frac{(2y)^2 - y^2 - 3}{2y - 2} = \lim_{(x,y)\to(2,1)} \frac{3y^2 - 3}{2y - 2} = \lim_{(x,y)\to(2,1)} \frac{3(y+1)(y-1)}{2(y-1)} = 3. \text{ Next let } y=1. \text{ Then } \]

\[ \lim_{(x,y)\to(2,1)} \frac{x^2 - 1 - 3}{x - 2} = 4. \text{ Thus the limit does not exist since there are two paths that produce different limits.} \]

28) implicit differentiation: \( y \) is a constant, \( x \) as usual, multiply by \( \frac{\partial z}{\partial x} \) after differentiating with respect to \( z \). \( \rightarrow ye^{x} \frac{\partial z}{\partial x} + \frac{2x+y}{x^2 + y^2} = 0 \rightarrow \frac{\partial z}{\partial x} = -\frac{2x+y}{ye^{x}(x^2 + y^2)} \)

29) \( f(x, y, z) = 2 \rightarrow \sqrt{x^2 + y^2 - z} = 2 \rightarrow z = x^2 + y^2 - 4 \)
This is a paraboloid shifted 4 units down from the origin.

30)

Through 1, since \( x=1 \),

\[ \lim_{(x,y)\to(1,-1)} \frac{x^2 + y}{x + y^2 - 2} = \lim_{(x,y)\to(1,-1)} \frac{1 + y}{1 + y^2 - 2} = \lim_{(x,y)\to(1,-1)} \frac{1}{y - 1} = \frac{1}{2} \]

Through 2, since \( y=-1 \),

\[ \lim_{(x,y)\to(2,7)} \frac{x^2 + y}{x + y^2 - 2} = \lim_{(x,y)\to(2,7)} \frac{x^2 - 1}{x + (-1)^2 - 2} = \lim_{(x,y)\to(2,7)} (x+1) = 2 \]

Thus the limit does not exist.

31) Switch \( x \) and \( y \):

\[ \int_{0}^{\frac{\pi}{4}} \int_{0}^{\frac{\pi}{4}} e^{-x^2} \, dx \, dy = \frac{1}{8} \left( \frac{1}{e^{16}} - 1 \right) \]

32) First find all the interior critical points by setting \( f_x = 0, f_y = 0 \). We get

\[ 2x - 4y = 0, -4x + 3y^2 + 4 = 0 \]

Take the first equation and solve for \( x \): then substitute it into the second. The interior critical points are \((4,2), (4/3, 2/3)\). Then find the critical points on the boundary. The first section is \( y = x \). Substitute \( y \) into \( x \) and we get

\[ g(x) = x^2 - 4x^2 + x^3 + 4x = x^3 - 3x^2 + 4x \]. Compute \( g'(x) \) and set it equal 0. But \( 3x^2 - 6x + 4 \) has no real solutions. The second boundary is \( y = -1 \). The equation becomes \( g(x) = x^2 + 4x - 5 \). Compute \( g'(x) \) and set it equal 0. \((-2,1)\) is a critical point. But this is not in the domain. On the third boundary, \( x = 7 \). The equation becomes \( g(y) = y^3 - 24y + 49 \). Compute \( g'(y) \) and set it equal 0. \((7,2\sqrt{2}),(7,-2\sqrt{2})\) are additional critical point. But \((7,-2\sqrt{2})\) is not in the domain. Now check the values of \( f(x,y) \) at the critical points \((4,2),(4/3, 2/3),(7,2\sqrt{2})\) and the corner points \((-1,-1),(7,-1),(7,7)\). Max = \( f(7,7) = 224 \), min = \( f(7,2\sqrt{2}) = -13.225 \)
33) First find the optimization function: \( f(x, y, z) = 8xyz \) since its sides are \( 2x, 2y, 2z \).

\(< f_x, f_y, f_z >= \lambda < g_x, g_y, g_z \) \( \rightarrow 8yz = 18x\lambda, 8xz = 72y\lambda, 8xy = 8z\lambda \). Since \( x, y, z \) are not 0, we can solve for \( \lambda \) without worrying about cases: we get \( \frac{8yz}{18x} = \lambda, \frac{8xz}{72y} = \lambda, \frac{8xy}{8z} = \lambda \) setting \( 1^{st} = 2^{nd} \)
gives \( y^2 = x^2/4 \) \( 1^{st} = 3^{rd} \) gives \( z^2 = (9/4)x^2 \). Now substitute this into the constraint to get

\[ 9x^2 + 9x^2 + 9x^2 = 36 \rightarrow x = \frac{2}{\sqrt{3}} (x \text{ cannot be negative}) \quad y = 1/\sqrt{3}, z = \sqrt{3} \]

34) A) The midpoints are (1,5), (3,5), (1,9), (3,9) and the area of each rectangle is 8.

\[ \int_{0}^{3} \int_{0}^{3} \ln y \, dy \, dx \approx 8((\ln 5) + (\ln 5) + (\ln 9) + (\ln 9)) \]

B) Integration by parts: \( u = \ln x, \, dv = dx \).

\[ \int_{0}^{3} \int_{0}^{3} \ln y \, dy \, dx = \int_{0}^{3} (y \ln y - y) \, dx = \int_{0}^{3} (11\ln 11 - 3\ln 3 - 8) \, dx = 3(11\ln 11 - 3\ln 3 - 8) \]

35) A) The region is a quarter of a circle in the third quadrant.

\[ \int_{0}^{\pi} \int_{0}^{r} \frac{1}{2} r^2 \, dr \, d\theta = \left( \int_{0}^{r} \frac{1}{2} r^2 \, dr \right) \left( \int_{0}^{\pi} d\theta \right) = \frac{1}{2} (e - 1)(\frac{\pi}{2}) \]

36) \(< f_x, f_y, f_z >= \lambda < g_x, g_y, g_z \) \( \rightarrow yz = 2x\lambda, xz = 4y\lambda, xy = 6z\lambda \). Case 1) if \( x, y, z \) are not 0, we can solve for \( \lambda \) without worrying about cases: \( \frac{yz}{2x} = \lambda, \frac{xz}{4y} = \lambda, \frac{xy}{6z} = \lambda \) setting \( 1^{st} = 2^{nd} \) gives \( y^2 = x^2/2 \)

\( 1^{st} = 3^{rd} \) gives \( z^2 = (1/3)x^2 \). Now substitute this into the constraint to get

\( x^2 + x^2 + x^2 = 6 \rightarrow x = \pm \sqrt{2}, y = \pm 1, z = \pm \sqrt{6/3} \) (there are 8 points) Case 2) If one or more variable is 0 then \( f(x, y, z) = xyz = 0 \). Combining the two cases, max is obtained when there are an even number of negative numbers (so the product is positive), min when the number of negatives is odd: max \( (\sqrt{2})(\frac{\sqrt{6}}{3}), \text{ min } - (\sqrt{2})(\frac{\sqrt{6}}{3}) \)

37) The right upper corner points are (0,1), (0,2), (1,1), (1,2) and the area of each rectangle is 1.

\[ \int_{0}^{1} \int_{0}^{1} (3x + 4y) \, dy \, dx \approx 1((3(0) + 4(1)) + (3(0) + 4(2)) + (3(1) + (4)(1)) + (3(1) + 4(2))) \]

38) The surface area is \( s = 2\pi rh: \frac{ds}{s} \cdot 100 = \frac{2\pi rh + 2\pi dh}{2\pi h} \cdot 100 = \frac{1}{r} (dr + \frac{1}{h} dh) \cdot 100 = (1\%) + (3\%) = 4\% \)

39) First find the critical points, then classify them using the criteria of 1) local max if \( f_{xx} < 0, D = f_{xx}f_{yy} - (f_{xy})^2 > 0 \) 2) local min if \( f_{xx} > 0, D = f_{xx}f_{yy} - (f_{xy})^2 > 0 \) 3) saddle if \( D = f_{xx}f_{yy} - (f_{xy})^2 < 0 \). For the critical points, solve \( f_x = 0, f_y = 0 \). We get
\[-x^2 + y = 0, x + y - 12 = 0\]  Take the first equation and solve for $y$. Then substitute into the second equation and solve for $x$: $x = 3, -4$. Substitute into the second to solve for $y$. The critical points $(3, 9), (-4, 16)$. $f_{xx} = -2x, f_{xy} = 1, f_{yy} = 1$ (3,9) is a saddle point, (-4,16) is a local min.

40) $f_x = \frac{1}{x+y}, f_y = \frac{1}{x+y}, f_{xx} = -\frac{1}{(x+y)^2}, f_{xy} = -\frac{1}{(x+y)^2}, f_{yy} = -\frac{1}{(x+y)^2}$. The linearization is $L(x, y) = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2) = \ln 3 + \frac{1}{3}(x-1) + \frac{1}{3}(y-2)$ b) First observe that $z = r$ is a cone since $z = r = \sqrt{x^2 + y^2}$. $z = \sqrt{2-r^2}$ is a sphere of radius $\sqrt{2}$ since $z = \sqrt{2-r^2} \rightarrow z = \sqrt{2-x^2 - y^2} \rightarrow x^2 + y^2 + z^2 = (\sqrt{2})^3$

\[
\int_0^{\frac{2\pi}{3}} \int_0^{\sqrt{r^2-r^2}} r \, dz \, dr \, d\theta \text{ in the spherical coordinates is } \int_0^{\frac{2\pi}{3}} \int_0^\frac{\pi}{4} \int_0^{\sqrt{3}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta
\]

41) $<f_x, f_y, f_z> = \lambda <g_x, g_y, g_z> \rightarrow 2x = 4x^3 \lambda, 2y = 4y^3 \lambda, 2z = 4z^3 \lambda$. Case 1) if $x, y, z$ are not 0, we can solve for $\lambda$ since we are not dividing by 0. We get \[\frac{2x}{4x^3} = \lambda, \frac{2y}{4y^3} = \lambda, \frac{2z}{4z^3} = \lambda\] Setting 1st = 2nd gives $y^2 = x^2$. 1st = 3rd gives $z^2 = x^2$. Now substitute this into the constraint to get $x^4 + x^4 + x^4 = 1 \rightarrow x = \pm \sqrt[4]{\frac{3}{2}}, y = \pm \sqrt[4]{\frac{3}{2}}, z = \pm \sqrt[4]{\frac{3}{2}}$ (there are 8 points) In this case, by going back to the optimization function, the values are $\sqrt{3}$ Case 2) If one or more variable is 0. Since both the optimization and constraint functions symmetric with respect to the three variables, we may assume $x = 0$. Setting 3rd = 2nd gives \[\frac{2y}{4y^3} = \lambda, \frac{2z}{4z^3} = \lambda \rightarrow y^2 = z^2\] Now substitute this into the constraint to get $0^4 + y^4 + y^4 = 1 \rightarrow x = 0, y = \pm \sqrt[4]{\frac{3}{2}}, z = \pm \sqrt[4]{\frac{3}{2}}$. This case produces the f-value $\sqrt{3}$, Case 3) When two of the variables equal 0: we may assume $x = y = 0$. This case go back to the constraint give $0^4 + 0^4 + z^4 = 1 \rightarrow x = 0, y = 0, z = \pm 1$, producing the f-value 1.

Therefore, the max is $\sqrt{3}$, the min is 1.
42) A) Observe that in polar, and \( y = x \rightarrow \theta = \frac{\pi}{4} \cdot \int_0^{\frac{\pi}{4}} (r \cos \theta) r \, dr \, d\theta \) B) The midpoints are (1, 1.5), (3, 1.5), (1, 4.5), (3, 4.5) and the area of each rectangle is 6. \( \int_0^4 \int_0^2 x \, dx \, dy = 8(1 + 1 + 3 + 3) = 64 \)

43) \( dv = 2\pi rh \, dr + \pi r^2 \, dh \). Since \( v = \pi r^2 h \) and \( \frac{dr}{r} \cdot 100 \leq \pm 1\% \), \( \frac{dr}{h} \cdot 100 \leq \pm 1\% \),

44) a. \( f_x = 2xe^y \, f_y = x^2 e^y \), \( f_{xx} = 2e^y \), \( f_{xy} = 2xe^y \), \( f_{yy} = x^2 e^y \). The linearization is \( L(x, y) = f(1, 0) + f_x(1, 0)(x-1) + f_y(1, 0)(y-0) = 2 + 2(x-1) + y = 2x + y \)

45) a. Use polar coordinates to compute \( \int_0^1 \int_{\sqrt{2-y^2}}^{1} \frac{1}{\sqrt{x^2 + y^2}} \, dx \, dy \)

First, observe that the right boundary \( x = \sqrt{2} \) is a circle

\[ \int_0^1 \int_{\sqrt{2-y^2}}^{1} \frac{1}{\sqrt{x^2 + y^2}} \, dx \, dy = \int_0^{\pi/4} \int_0^{\sqrt{2}} \frac{1}{r} \, r \, dr \, d\theta = \frac{\pi}{4} \]
46) \(<f_x, f_y, f_z> = \lambda <g_x, g_y, g_z> \rightarrow 3z = 2x\lambda, 6 = 4y\lambda, 3x = 2z\lambda\). By inspection, we can see that \(y\) cannot be 0 since \(y = 0\) would yield \(6 = 0\). Case 1) if \(x, y, z\) are not 0, we can solve for \(\lambda\) since we are not dividing by 0. We get \(\frac{3z}{2x} = \lambda, \frac{6}{4y} = \lambda, \frac{3x}{2z} = \lambda\) Setting \(1^{st} = 3^{rd}\) produces \(z^2 = x^2 \rightarrow z = \pm x\) \(1^{st} = 2^{nd}\) produces \(y = \frac{x}{z}\). But since \(z = \pm x, y = \pm 1\) Now substitute this into the constraint to get \(x^2 + 2(1)^2 + x^2 = 6 \rightarrow x = \pm \sqrt{2}, y = \pm 1, z = \pm \sqrt{2}\) (there are 8 points) In this case, by going back to the optimization function, the maximum value is 18 (when all the variables are positive) and the minimum is -18 when \(x\) and \(z\) have the opposite signs and \(y\) is negative. Case 2) \(x = 0\). In this case, since \(3z = 2x\lambda\), \(z\) is also 0. (\(We\ observed\ earlier\ that\ \(y\)\ cannot\ be\ 0\)). Going back to the constraint, \(0^2 + 2y^2 + 0^2 = 6 \rightarrow y = \pm \sqrt{3}\). This case produces the \(f\)-values \(\pm 6\sqrt{3}\). Combining the two cases, we see that the max is 18, min is -18.

47) Remember, \(dx\) ranges from the left equation to the right equation. On the left, \(x = \sqrt{y} \rightarrow y = x^2\). On the right, \(x = 2 - \sqrt{y} \rightarrow y = (2 - x)^2\). Thus the region looks like

![Diagram](image)

Using \(dy\) first, you must break up the region into two pieces: from 0 to 1, then from 1 to 2 since the equation changes at \(x = 1\).

\[
\int_0^1 \int_0^{x^2} 1 \, dy \, dx + \int_1^2 \int_0^{(2-x)^2} 1 \, dy \, dx
\]

48) It is given that \(\frac{dR_1}{R} = 0.01, \frac{dR_2}{R} = 0.02, R_1 = 100, R_2 = 200\). We are seeking \(\frac{dR}{R} \cdot 100\). First find \(R\):

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}, R_1 = 100, R_2 = 200 \rightarrow R = \frac{200}{3}.
\]

Differentiate implicitly, we get
Now going back to the constraint, $x + yz = 5$, we have $x = 5$, obtaining $(5,0,0)$. Case 2) $\lambda = 2$. $2x = \lambda \rightarrow 2x = 2 \rightarrow x = 1$. $2y = \lambda z \rightarrow 2y = 2z \rightarrow y = z$. Now going back to the constraint, $x + yz = 5 \rightarrow 1 + y^2 = 5 \rightarrow y = z = \pm 2$, obtaining $(1,2,2)$ and $(1,-2,2)$ . Case 3) $\lambda = -2$. $2x = \lambda \rightarrow 2x = -2 \rightarrow x = -1$. $2y = \lambda z \rightarrow 2y = -2z \rightarrow y = -z$. Now going back to the constraint, $x + yz = 5 \rightarrow -1 - y^2 = 5 \rightarrow y^2 = -6$, which has no solution. Thus the last case is impossible,
Thus the minimum is 9.

Method 2) (This method is a bit longer but easier in many cases)

\[ <f_x, f_y, f_z> = \lambda <g_x, g_y, g_z> \Rightarrow 2x = \lambda, 2y = \lambda z, 2z = \lambda y \]

In this method, solve for \( \lambda \) and use the equations to eliminate other variables using cases.

Case 1) \( z \neq 0 \).

This case, since by the third equation \( 2z = \lambda y \rightarrow y \neq 0 \)

Using the second and the third, \( 2y = \lambda x \rightarrow \lambda = \frac{2y}{x} \). \( 2z = \lambda y \rightarrow \lambda = \frac{2z}{y} \).

By setting them equal to each other, we have \( \frac{2y}{x} = \frac{2z}{y} \rightarrow y^2 = \frac{x^2}{y} \rightarrow y = \pm z \). Therefore, since \( \lambda = \frac{2z}{y} \), we get \( \lambda = \frac{2z}{y} = 2 \) or \( -2 \) by going back the first constraint \( 2x = \lambda \rightarrow 2x = \pm 2, \rightarrow x = \pm 1 \)

Now go back to the constraint \( x + yz = 5 \). We can see that since \( x = 1 \), \( y = -z \) results in \( -1 + (-z)(z) = 5 \rightarrow z = -4 \), which is impossible. Thus \( x = 1 \) and \( y = z \): \( 1 + (z)(z) = 4 \rightarrow z = 2, -2 \), obtaining two ordered pairs \((1,2,2)\) and \((1,-2,-2)\)

Case 2) \( z = 0 \)

By looking at the third equation, \( y = 0 \) and \( 2z = \lambda y \rightarrow 2z = 0 \rightarrow z = 0 \). Now use \( y = 0 \), \( z = 0 \) and the constraint \( x + yz = 5 \), we have \( x = 5 \), obtaining \((5,0,0)\)

Now we can make the same table as the method 1.

\[ 52 \]

A) Write \( \iiint_{0 \leq z \leq 2-x} \int_{0}^{2-x} \int_{0}^{y^2-z^2} dz \ dy \ dx \) using \( dy \ dz \ dx \)
\[
\int_0^2 \int_0^{2-x} \int_0^{9-x^2} \,dz\,dy\,dx = \int_0^2 \int_0^{2-x} \int_0^{9-x^2} \,dy\,dz\,dx
\]

b) \[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 r^3 \,dr\,d\theta = \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 + y^2) \,dy\,dx
\]

\[
f_x = -\frac{2x}{(x^2 + y^2)^2}, \quad f_y = -\frac{2y}{(x^2 + y^2)^2}, \quad f_{xx} = \frac{6x^2 - 2y^2}{(x^2 + y^2)^3}, \quad f_{yy} = \frac{6y^2 - 2x^2}{(x^2 + y^2)^3}, \quad f_{xy} = \frac{8xy}{(x^2 + y^2)^3}
\]

53) a) Linearization: \( f(1,1) = \frac{1}{2} \), \( f_x(1,1) = -\frac{1}{2} \), \( f_y(1,1) = -\frac{1}{2} \). Thus \( L(x, y) = \frac{1}{2} - \frac{1}{2} (x-1) - \frac{1}{2} (y-1) \)

54) Place the cylinder on the xy-plane. Then the line through the center of mass is the z-axis. Since \( x^2 + y^2 = r^2 \), \( I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^R \int_0^h r^2 \,dz\,dr\,d\theta = (\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \,d\theta) (\int_0^R r^3 \,dr) (\int_0^h \,dz) = \frac{\pi(R_2^4 - R_1^4)h}{2} \). Then to make match \( I = \frac{1}{2} M (R_1^2 + R_2^2) \) \( \frac{\pi(R_2^4 - R_1^4)h}{2} = \frac{1}{2} \pi(R_2^2 - R_1^2)h(R_2^2 + R_1^2) = \frac{1}{2} M (R_2^2 + R_1^2) \)

55)
\[ \iint_{0}^{\pi/2} \int_{0}^{\pi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \]

56) **A)** This is a paraboloid lying over the SECOND quadrant since the x-values are negative and the y-values are positive.

\[ \int_{0}^{\pi/4} \int_{0}^{\pi} r^3 \, dz \, dr \, d\theta \]

**B)** This is the right half of a sphere since x is positive and y can be positive or negative.

\[ \int_{0}^{\pi/2} \int_{0}^{\pi/2} \rho^5 \sin \phi \, d\rho \, d\phi \, d\theta \]

57)

58) **A)** Observe that \( z = 2 \rightarrow \rho \cos \phi = 2 \rightarrow \rho = \frac{2 \sec \phi}{\cos \phi} \)

\[ \iiint_{0}^{\pi/2} \int_{0}^{\pi} (\rho \cos \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \]

**B)** \( \iiint_{0}^{\pi/2} \int_{0}^{\pi} \rho \, d\rho \, d\phi \, d\theta \)

59) **A)** First observe that in spherical, \( z = 1 \rightarrow \rho \cos \phi = 1 \rightarrow \rho = \sec \phi \). To find the range of \( \phi \), sketch the two-dimensional cross section and observe that the cone and \( z = 1 \) makes 30-60-90 triangle.

\[ \iiint_{0}^{\pi/3} \int_{0}^{\pi/2} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \]

**b)** To find the projection, square both sides to get \( x^2 + y^2 = z^2 \) and substitute into the second equation, obtaining \( z^2 = z - 2 \rightarrow z = 1 \). So the projection is the unit circle.

\[ I_z = \int_{0}^{\pi} \int_{0}^{\pi/2} \rho^2 \, r \, dz \, dr \, d\theta \]
60) First sketch the region of
\[
\int \int \int_{-1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \int_{-1+\sqrt{1-(x^2+y^2)}}^{1+\sqrt{1+(x^2+y^2)}} (x^2 + y^2 + z^2)^{\frac{3}{2}} \, dz \, dy \, dx
\]

Observe that
\[z = 1 - \sqrt{1-(x^2+y^2)} \rightarrow z = 1 - \sqrt{1-(x^2+y^2)} \rightarrow (z - 1)^2 = 1 - x^2 - y^2 \rightarrow x^2 + y^2 + (z - 1)^2 = 1. \text{ This is a sphere of radius } 1 \text{ centered at (0,0,1). Thus the region looks like}\]

\[\text{projection}\]

\[\text{projection}\]

a) In cylindrical, of
\[
\int \int \int_{0}^{2\pi} \int_{0}^{1} \int_{0}^{1+\sqrt{1+r^2}} (r^2 + z^2)^{\frac{3}{2}} \, r \, dz \, dr \, d\theta
\]

b) In spherical, by looking at the sketch, you see that \(\rho\) ranges from the origin to the top of the sphere:

In spherical coordinates, \(x^2 + y^2 + z^2 - 2z + 1 = 1 \rightarrow x^2 + y^2 + z^2 = 2z \rightarrow \rho^2 = 2\rho \cos \phi \rightarrow \rho = 2 \cos \phi\)
Thus \[ \int_0^2 \rho \int_0^\pi \int_0^2 \cos \phi \left( \rho^2 \right) \frac{3}{2} \left( \rho^2 \sin \phi \right) d\rho \ d\phi \ d\theta \]

61) By placing the base on the x-axis, this becomes the problem of computing \( I_x \) (see the picture below). It is easier to set up the integral using \( dx \ dy \). You would need to break up the integral into two parts if you use \( dy \ dx \).

![Diagram](image)

You can find the side equations using \( y = mx+b \) formula. Then you solve for \( x \) since you are using \( dx \ dy \).

\[ I_x = \int_0^{10} \int_{\frac{4y+10}{4}}^{\frac{10-y}{4}} y^2 \ dx \ dy \]

62) It is best to use the z-axis as the diameter for which the moment of inertia is computed. This is because \( x^2 + y^2 \) simplifies nicely as \( (\rho \sin \phi \cos \theta)^2 + (\rho \sin \phi \sin \theta)^2 = \rho^2 \sin^2 \phi \). Thus using the spherical coordinates, \( I_z = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{\rho^3} \rho d\rho d\theta \).

Note that \( \int_0^{\pi} \sin^3 \phi \ d\phi = \int_0^{\pi} \sin \phi \sin \phi \ d\phi = \int_0^{\pi} \sin \phi (1 - \cos^2 \phi) \ d\phi = -\int_1^1 (1 - u^2) du = \frac{4}{3} \)

Thus \( I_z = (2\pi) \left( \frac{4}{3} \right) \left( \frac{1040}{4} \right) = \frac{2080\pi}{3} \)

63) Write \( \int_0^2 \int_0^{\frac{2-x^2}{2-x}} dz \ dy \ dx \) using \( dy \ dz \ dx \)
\[ \int_0^2 \int_0^{9-x^2} \int_0^{2-x} dydzdx = \int_0^2 \int_0^{9-x^2} dydzdx \]

B) (5 points) Write \( \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^r dzdrd\theta \) using the spherical coordinates.

First observe that 
\[ z = 1 \rightarrow \rho \cos \phi = 1 \rightarrow \rho = \frac{1}{\cos \phi} \]

Thus
\[ \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^r dzdrd\theta = \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^{\frac{1}{\cos \phi}} \rho^2 \sin \phi \ d\rho \ d\phi \ d\theta \]

\[ V = \int_0^1 \int_x^1 \int_0^{\sqrt{z-x^2}} ldydzdx \]

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First observe that the line of intersection of the planes \( x + z = 1 \) and \( z = 2x \) becomes one of the boundary lines of the projection. Eliminate \( x \) from the above equations, we obtain \( z = 1 - x = 1 - 2y \)

\[
V = \int_{0}^{1} \int_{-2y}^{1} \int_{0}^{1} 1 \, dx \, dz \, dy
\]

66) A) Does \( \lim_{(x,y) \to (0,0)} \frac{x^4}{x^2 + 2x^2y^2} \) exist? Justify your answer.

Yes, use the squeeze theorem:

\[
0 \leq \frac{x^4}{x^2 + 2x^2y^2} \leq x^2 \quad \text{and} \quad \lim_{(x,y) \to (0,0)} x^2 = 0.
\]

Thus \( \lim_{(x,y) \to (0,0)} \frac{x^4}{x^2 + 2x^2y^2} = 0 \)

b) Does \( \lim_{(x,y) \to (2,1)} \frac{x + 2y - 4}{x^2 + 2y^2 - 6} \) exist? Justify your answer.

No. Find two paths that produce different limits.

First let \( x = 2 \). Then

\[
\lim_{(x,y) \to (2,1)} \frac{x + 2y - 4}{x^2 + 2y^2 - 6} = \lim_{(x,y) \to (2,1)} \frac{2 + 2y - 4}{2^2 + 2y^2 - 6} = \lim_{(x,y) \to (2,1)} \frac{2(y - 1)}{2(y + 1)(y - 1)} = \frac{1}{2}
\]

Next let \( y = 1 \). Then

\[
\lim_{(x,y) \to (2,1)} \frac{x + 2y - 4}{x^2 + 2y^2 - 6} = \lim_{(x,y) \to (2,1)} \frac{x + 2 - 4}{x^2 + 2(1)^2 - 6} = \lim_{(x,y) \to (2,1)} \frac{(x - 2)}{(x + 2)(x - 2)} = \frac{1}{4}
\]

67) A)

Place the hypotenuse onto the y-axis as shown below. Then \( A = (2\sqrt{2}, 2\sqrt{2}) \)

\[
I_y = \int_{0}^{2\sqrt{2}} \int_{-\sqrt{2}}^{\sqrt{2}} x^2 \, dy \, dx = \int_{0}^{2\sqrt{2}} x^2 (4\sqrt{2} - 2x) \, dx = \frac{32}{3}
\]

68) Compute the centroid of the solid bounded above by the cone \( z = \sqrt{x^2 + y^2} \), side by the sphere of radius 2, below by the xy-plane.
By symmetry, \( \bar{x} = \bar{y} = 0 \). To use the spherical coordinates to find \( \bar{z} \).

\[
M = \int_0^{2\pi} \int_0^\pi \int_0^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = (\int_0^2 \rho^2 d\rho)(\int_0^\pi \sin \phi \, d\phi)(\int_0^{2\pi} d\theta) = \frac{8\sqrt{2}}{3} \pi
\]

\[
M_{xy} = \int_0^{2\pi} \int_0^\pi \int_0^2 (\rho \cos \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = (\int_0^2 \rho^3 d\rho)(\int_0^\pi \sin \phi \, d\phi)(\int_0^{2\pi} d\theta) = 2\pi
\]

\[
\bar{z} = \frac{M_{xy}}{M} = \frac{2\pi}{\frac{8\sqrt{2}}{3} \pi} = \frac{3}{4\sqrt{2}}. \quad \text{Thus the centroid is } (0,0,\frac{3}{4\sqrt{2}})
\]

69)

a)

Note that \( y=x \) is a 45 degree line on the right side. The top boundary is \( y = 1 \rightarrow r \sin \theta = 1 \rightarrow y = \csc \theta \)

\[
\int_0^{\frac{\pi}{4}} \int_0^{\csc} r \, dr \, d\theta
\]

b) This is a sphere of radius 2 in the first octant.

\[
\int_0^{\frac{\pi}{2}} \int_0^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta =
\]

c) Two integrals are required since the top boundary equation changes at \( x=0 \).
70) A) true: if the gradient is the zero vector, then \( f_x = 0 \) and \( f_y = 0 \)  
b) true: the normal line and the gradient are both perpendicular to the surface.  
C) about BC is greater since its weight is more dispersed about BC. It is much harder to spin the plate about BC, requiring greater kinetic energy.